GMT AO System Requirements and Error Budgets in the Preliminary Design Phase

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ABSTRACT

Error budgets are an indispensable tool for assuring that project requirements can be and are being met. An error budget will typically include terms associated with subsystems which are being designed by different teams of engineers, and fabricated by different vendors. It is a useful tool at all levels of design since it provides a means to negotiate design trades in the broadest possible context. Error budgeting is in many ways fundamental to the mission of systems engineering and of course to the overall project success.

In this paper we will describe the GMT Adaptive Optics System flow down requirements and their integration with their wavefront error budgets. We will focus on the GMT Adaptive Optics wavefront error budgets for the following observing modes: Natural Guide Star Adaptive, Laser Tomography Adaptive Optics and Ground Layer Adaptive Optics. Finally, a description of the error budgets and the close link between the error budgets and other parameter such as sky coverage, zenith angle, etc., will be discussed in this paper.

Keywords: LTAO, NGS AO, AO, GLAO, Error budget, Requirements

1. INTRODUCTION

The Adaptive Optics (AO) System will be an integral part of the GMT telescope, providing guide star generation, wavefront sensing, and wavefront correction for many of the currently planned science instruments. The system will use natural and laser guidestars to sense phase errors induced by index of refraction variations in the earth’s atmosphere.

The AO System will use an Adaptive Secondary Mirror (ASM) to correct disturbances caused by variations of the index of refraction across the pupil integrated along the line of sight through the atmosphere, and slowly varying telescope and instrument-caused wavefront errors. In some observing modes, separate image stabilization and segment phasing subsystems will provide additional input to the ASM control.

An overview of the GMT project and GMT AO System are in Johns [1] and Bouchez [2] respectively.

The AO System will provide several observing modes to science instruments in a way, which maximizes the scientific productivity of the science instruments while minimizing replication of functionality within the AO System. The AO observing modes are specified by the AO requirements herein.

The capabilities are grouped as follows and they are illustrated schematically in Figure 1:

- Natural Guide Star AO (NGSAO): The NGS AO observing mode uses a single natural guidestar (NGS) wavefront sensor to provide all of the wavefront correction information for the AO System. The wavefront aberration will be compensated by the ASM, providing diffraction-limited imaging at 0.9-25 µm wavelength over a field of view limited by atmospheric anisoplanatism.

- Laser Tomography AO (LTAO): The LTAO observing mode uses a ~1 arcmin diameter constellation of Laser Guidestars (LGS) to tomographically reconstruct the high-order components of the atmospheric wavefront aberrations in the direction of a central science target. One or more faint natural guidestars must be used to measure tip-tilt, focus, segment piston, and dynamic calibration terms. The wavefront aberration will be compensated by the ASM, providing diffraction-limited imaging at 0.9-25 µm wavelength over a field of view limited by atmospheric anisoplanatism.
Ground Layer AO (GLAO): The GLAO observing mode uses a guidestar asterism (either LGS/NGS or NGS-only) to detect and correct wavefront errors common to sky objects within a large (up to 10 arcmin in diameter) field of view. These errors are mainly due to low (up to 1 km) altitude components of the atmospheric wavefront aberrations. The wavefront aberration will be detected using multiple wavefront sensors and compensated by the ASM, resulting in improved natural seeing images over a field of view comparable to the guidestar constellation size. While providing some improvement in the visible, GLAO correction is expected to be particularly useful at wavelengths longer than 1 μm.

Figure 1: Schematic illustrations of Single Conjugate (Natural Guidestar) AO, Laser Tomography AO, and Ground Layer AO. Note that the GMT GLAO design can use lasers and/or natural reference stars. Image credit: E. Marchetti, ESO.

2. GMT ADAPTIVE OPTICS SCIENCE REQUIREMENTS

2.1. Natural Guidestar AO Observing Mode

The NGSAO performance requirements are defined for median integrated turbulence and wind speed, and a typical turbulence profile (see section 4.2 in Maiten [3]), 15° from zenith. The performance of the NGSAO mode is specified by the following primary science requirements:

SCI-1882: The GMT in NGSAO mode, together with an appropriate coronagraphic instrument, shall deliver a contrast of no less than $10^5$ at 122 mas from a bright guide star in the L' band (3.77 μm), over a 3600s integration composed of shorter exposures (goal: $10^6$ contrast at 61 mas separation in the L' band).

SCI-1883: The GMT in NGSAO observing mode shall deliver an on-axis K band (2.2 μm) Strehl ratio of no less than 0.75 when using an V=8.0 G2V guide star, over a 120s integration.

2.2. Laser Tomography AO Observing Mode

The LTAO performance requirements are specified for median integrated turbulence and wind speed, a typical turbulence profile, and seasonal minimum mesospheric sodium layer density, 15 degrees from zenith. The performance of the LTAO mode is specified by the following requirements:

SCI-1884: The GMT in LTAO observing mode shall deliver an on-axis H band (1.65 μm) Strehl ratio of no less than 0.3 over at least 20% of the sky at the galactic pole, over a 120 s integration (goal: 50% at the galactic pole).

SCI-1885: The GMT in LTAO observing mode shall deliver a K band (2.2 μm) fractional ensquared energy in 50x50 mas of no less than 0.4 over at least 50% of the sky at the galactic pole, over a 900 s integration (goal: 90% of the sky at the galactic pole).

SCI-1886: The GMT in LTAO observing mode shall deliver a K band (2.2 μm) fractional ensquared energy in 85x85 mas of no less than 0.5, and a PSF FWHM no greater than 20 mas, when using an on-axis K=15 guidestar, over a 900 s integration.
2.3. Ground Layer AO Observing Mode

The GLAO performance will be a strong function of the vertical distribution of turbulence, which cannot easily be described by a single parameter. The degree of image quality improvement is sensitive to the vertical distribution of turbulence. GLAO performance is therefore defined in terms of the probability of achieving an image FWHM over many nights, in seasonal minimum mesospheric sodium density conditions, 15° from zenith.

SCI-1887: The GMT in GLAO observing mode shall deliver to the instrument focal plane a K band (2.2 µm) point spread function FWHM less than 0.3 arcsec (goal: 0.25 arcsec), averaged over a 6.5 arcmin diameter field of view, with a 50% of probability.

SCI-4509: The GMT in GLAO observing mode should deliver a reduction in the I band (790 nm) point spread function FWHM of 15% with respect to the seeing-limited value, averaged over 6.5 arcmin over field of view, with at least 50% probability.

3. ERROR BUDGET AND PERFORMANCE CONSIDERATION

The science requirements for the GMT AO System are specified for median integrated turbulence and wind speed, a typical turbulence profile, and seasonal minimum mesospheric sodium layer density, 15 degrees from zenith. This has the great benefit of setting a single set of conditions for all simulations and analysis and thereby enables design options to be compared without the confusion that variation in the parameters can sometimes cause. It does not, however, tell us how the system will perform in best or worst conditions.

Prior to establishing an error budget for each AO mode, we therefore performed certain simulations designed to estimate the performance of a baseline AO system which just meets the Science Requirements, over a wide range of conditions. These simulations combine simple analytic formulae for wavefront error term, anchored by the results of Monte-Carlo simulations performed using the Yorick Adaptive Optics (YAO) simulation code developed by Francois Rigaut and extended for the GMT by Marcos van Dam [4].

The following abbreviations and assumptions are used: S_K indicates Strehl in the K band (2.19 µm), while EE25_J indicates the fractional of light falling in a 25 mas square aperture in the J band (1.22 µm). The Fried parameter r_0, inversely proportional to the integrated turbulence strength, is specified for 500 nm wavelength. It has a median value at Campanas Peak r_0=16.4 cm. unless otherwise noted, all estimates are for median seeing, 15° from zenith, in seasonal minimum sodium conditions.

3.1. NGSAO and LTAO Performance

We begin by comparing the Strehl ratio expected in the NGSAO and LTAO observing modes (Figure 2). The NGSAO curves are for an on-axis V=8 guidestar (SCI-1883), while the LTAO values are shown for the K=15 on-axis guidestar case (SCI-1886), and for 20/50/90% sky fractional coverage at the galactic pole (SCI-1884 and SCI-1885). The performance at a given fraction of sky coverage is determined through Monte Carlo simulations using randomly generated star fields. A sky coverage of 50% at the galactic pole corresponds to ~80% over the full sky accessible from Las Campanas.

Figure 2: NGSAO and LTAO compared. Strehl versus wavelength, as a function of: AO mode and guidestar magnitude or sky coverage fractions at the galactic pole (r_0=16.4 cm, z=15°, median conditions).
In addition to providing high Strehl throughout the near-infrared, the NGSAO mode will also deliver useful Strehl down to the I band ($S_{\text{I band}}=0.15$). The LTAO mode, in contrast, will only provide a useful diffraction-limited imaging capability at J band and longer, over ~20% of the sky at the galactic pole. For science targets (with fainter and/or more distant guidestars), only the K band and longer will offer a reasonable Strehl ($S_{\text{K band}}=0.1-0.2$) for imaging. However, it should be noted that these values are in median conditions, and that this performance is significantly better than any present LGSAO system on an 8-10m telescope.

The signal-to-noise ratio achieved on a point source with an imaging camera is roughly proportional to the Strehl ratio. However, the Strehl ratio is not a particularly useful performance metric for spectroscopic instruments where the slit width or spaxel size may be larger than the diffraction-limited image width. Such instruments are far less sensitive to residual tip-tilt error, while remaining sensitive to the high-order wavefront errors, which scatter light far from the PSF core. The fractional ensquared energy in an aperture of a given size is a more relevant metric for spectrographs and IFUs.

Figure 3 compares the NGSAO and LTAO performance in terms of ensquared energy, at J and K bands. In the J band, the NGSAO mode (using bright guidestars) provides EE>0.15 in any sized aperture up to 85 mas. The fraction of energy in the PSF core is simply too low in the LTAO mode to provide good energy concentration at J band. However, the situation is quite different at K band. There the LTAO mode, even at 90% sky coverage at the pole, can deliver EE25K>0.25 and EE50K>0.40. Thus the effective sky coverage for K band spectroscopy with 50 mas apertures is nearly complete. The H band performance falls midway between these two cases.

![Figure 3](image_url)

Figure 3: Ensquared energy versus aperture size in the J and K bands, as a function of AO mode and guidestar magnitude or sky coverage fractions at the galactic pole ($r_0=16.4\text{ cm, } z=15^\circ$).

Figure 4 (Top) illustrates how the NGSAO performance varies with guidestar magnitude, while Figure 4 (Bottom) provides the same information for the LTAO observing mode. In median conditions, the limiting magnitude of the NGSAO mode will be approximately $V=13$. The LTAO mode, when using on-axis guidestars, has a limiting magnitude of $K=19$, with a rapid loss of tip-tilt control beyond this.

Figure 5 and Figure 6 demonstrate how NGSAO and LTAO performance varies with seeing and zenith angle. In particularly good seeing (10th percentile, $r_0=24.6\text{ cm}$), the baseline NGSAO System is capable of delivering ~150 nm RMS wavefront error when using bright guidestars. This particularly benefits observations at short wavelengths, improving the J Strehl to $S_{J}=0.52$, and the ensquared energy to EE25J=0.42 (not shown).

The right panel of Figure 5 illustrates that the NGSAO performance is quite sensitive to zenith angle, and performs best above $z<30^\circ$.

The LTAO mode is even more sensitive to both seeing and zenith angle, due to the double-pass of the laser light through the turbulence and the reduced brightness of the LGS at higher zenith angles. In 10th percentile seeing conditions, the LTAO mode is capable of delivering ~205 nm RMS high-order wavefront error over 20% of the sky at the galactic pole.
Figure 4: (Top Left) NGSAO Strehl versus guidestar magnitude. (Top Right) NGSAO Ensquared energy versus guidestar magnitude. (Bottom Left) LTAO Strehl versus NGS magnitude, when using an on-axis infrared guidestar. (Bottom Right) LTAO ensquared energy versus NGS magnitude, when using an on-axis infrared guidestar.

Figure 5: NGSAO with V=8 guidestar. (Left) NGSAO Strehl versus Fried parameter $r_0$, when observed at $z=15^\circ$. The values indicated on the x-axis correspond to 90/75/50/25/10 percentiles. (Right) NGSAO Strehl versus zenith angle, when $r_0=16.4$ cm.

Figure 6: LTAO at 20% sky coverage. (Top Left) K Strehl versus Fried parameter $r_0$. (Top Right) K Strehl versus zenith angle. (Bottom Left) Ensquared energy at K versus $r_0$. (Bottom Right) Ensquared energy at K versus zenith angle.
### 3.2. GLAO Performance

The performance of the system in GLAO mode can be estimated by an analytic model of the PSF, as described by Tokovin [4]. The structure function is calculated for a given turbulence profile, and a given guide star constellation. Turbulent layers conjugated further from the deformable mirror have less of their structure function corrected, and thus contribute more to the image width.

The modeling takes into account the finite size of the wavefront sensor subapertures, which we have set at 0.65 m for these calculations. We have chosen a guide star constellation size of 7 arcmin diameter. The analytical modeling does not model the servo lag or the wavefront measurement error of the system.

We are using the synthetic profiles compiled by Goodwin [6] in his thesis originating from his two campaigns at Las Campanas in September 2007, and January 2008. These runs showed very distinct contributions to the seeing from the ground-layer. 18 profiles are available which permute “Good”, “Typical”, and “Bad” values for the ground layer and the free atmosphere. While broader statistics are needed, these initial profiles can be used to show expected performance.

Figure 7 (Left) shows a histogram of the K band FWHM values from the 18 profiles of the Goodwin models [6] appropriately weighted for their relative probability of occurrence. The average FWHM values in open and close loop are 0.5 and 0.19 arcsec respectively. The average improvement is a factor of 2.9, which is more than the ratio of these two values, since worse seeing results provide larger improvements.

Figure 7 (Right) shows a histogram of the I band FWHM values from the same profiles. The open and closed loop at I band are 0.61 and 0.41 arcsec respectively. The factor of improvement is 1.54.

In summary, it is expected that the GLAO System will provide a FWHM<sub>k</sub>&lt;0.30 arcsec images across a 6.5 arcmin diameter field of view at least 50% of the time, with image size near the FWHM<sub>k</sub>&lt;0.25 arcsec goal at the field center, if desired. On strong ground layer nights, the system should deliver images as small as FWHM<sub>k</sub>&lt;~0.10-0.15 arcsec.

These initial results suggest that GLAO is quite useful at NIR wavelengths, while being more modest for wavelengths shortward of 1 µm. However, the amount of data these distributions are based upon is rather small, and distinctly bimodal: one run had significant ground-layer while a second run had a distinct lack of it. Additional turbulence profiling, especially for the lowest 1 km of the atmosphere, will be crucial for refining the performance of the GMT GLAO System.
4. GMT ADAPTIVE OPTICS SYSTEM TOP-DOWN ERROR BUDGET

Error budgets are essential tools for evaluating and tracking the various factors in the Adaptive Optics design that may degrade performance.

The GMT AO error budgets serve three purposes:

1. Errors are allocated between various sources in order that the total contribution satisfies the performance specifications set forth in the Science Requirements. As the GMT design matures, error budget line items can be reallocated to provide relief in some areas at the expense of others in order to maintain specified performance. This reduces the total system cost and risk. The error budgets are dynamic tools that will be periodically updated throughout the life of the project.

2. The output of the error budget is an estimation of AO System performance and is the input for the design of science instruments.

3. The error budget establishes the system level requirements for the GMT systems. Sub-system requirements are derived (“flow down”) from these. Sub-systems are divided into component levels, which each have their own associated error.

The “top-down” error budgets reported in this section start with an overall specified system level of performance and allocated error allowances between subsystems.

4.1. LTAO and NGSAO Error budgets

For diffraction-limited AO System (NGSAO and LTAO), the error budget is defined in wavefront error expressed in rms nanometer. The Image Quality error budget for the NGSAO and LTAO presented here considers two main components: High order wavefront errors and Image Motions (tip-tilt) errors.

4.1.1. High Order Errors

- **AO High Order Error:**
  - Atmospheric fitting: The component of the wavefront that cannot be corrected by the DM due to the finite degrees of freedom of the DM. The fitting error depends on the spacing, location and influence functions of the actuators.
  - Temporal bandwidth: Results from the lag in the AO System's response to changes in the wavefront. The bandwidth error is a function of the frame rate; compute delays and the control law.
  - HO WFS measurement: The source of the noise errors is the uncertainty in the centroid estimates that is due to the finite number of photons on the WFS and the read-out noise of the WFS detector.
  - Atmospheric Piston: The component from the atmospheric phase errors, which lead to apparent piston difference across the GMT segments (not sensed by the Shack-Hartmann WFS).

- **Telescope Piston:** The GMT has a Gregorian optical design, with a primary mirror (M1) composed of seven 8.4m diameter circular segments, and an identically segmented secondary mirror (ASM). To achieve the diffraction limit of the full 25.4 m diameter aperture, the seven M1-ASM pairs must be phased such that their optical path length is matched to a small fraction of the observing wavelength. This is the term that takes into account the accuracy of that phasing.

- **AO Static Calibration:** These will be the contribution for the Non Common Path Aberration (NCPA), these include:
  - WFS optics
  - Instrument dichroic (reflection)
  - LGS dichroic (transmission)
  - Alignment (e.g. WFS-to-ASM)
  - Field-dependent aberrations

- **Uncorrectable Telescope Aberrations:** These are the contributions of the residual wavefront error that the telescope will be able to perform w/o the AO.

- **Uncorrectable Instrument Aberrations:** These are the contributions of the residual wavefront error that the instruments because of their optics and design will contribute to the overall image quality.
4.1.2. Image Motions Errors

- AO Tip/Tilt Errors: These include errors associated with tip-tilt measurement (photon noise, detector noise) and those due to the anisoplanatism suffered by off-axis guidestars.

The NGSAO requirements can be met with a system, which delivers <180 nm RMS high-order wavefront errors at the science detector, and <1.6 mas of residual tip-tilt error over short (120s) exposures.

The LTAO requirements require that the high-order wavefront error to be controlled to <260 nm RMS, and a residual tip-tilt error of 2.5 mas RMS when using an on-axis K=15 natural guidestar, <3 mas when using off-axis guidestars providing 20% sky coverage, and <6 mas at 50% sky coverage.

The most challenging aspects of the LTAO mode performance will likely be achieving <3 mas tip-tilt error using the faint NGS available at the 20% sky coverage level, in the presence of windshake and other sources of vibration.

Examples of LTAO and NGSAO error budgets for specific science requirements are shown in Figure 8 and Figure 9.

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<th>SMT AO Error Budget #</th>
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</thead>
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<td>Requirement</td>
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<td>Description</td>
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<tr>
<td>Conditions</td>
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### Table 1: LTAO Error Budget

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| Image motion |   |   |   |   |                 |
| AO-1762      |   |   |   |   |                 |
| Tip-tilt errors | max | 21 |   |   | AO-1762        |
| Tip-tilt measurement | max | 1.5 |   |   | AO-1762        |
| Tip-tilt temporal bandwidth | max | 0.5 |   |   | AO-1762        |
| Tip-tilt anisokinetism | max | 1.3 |   |   | AO-1762        |
| PSF FWHM       | max | 3.55 |   |   | SCI-1884       |

Figure 8: LTAO top-down error budget for SCI-1884 requirement.
### Figure 9: NGS AO top-down error Budget for SCI-1883 requirement.

<table>
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</table>

**Image motion**

| AO Tip-tilt errors | mas | 1.56 |  | AO-1764 |
| Tip-tilt measurement | mas | 1.56 |  |  |
| Tip-tilt temporal bandwidth | mas | 0.59 |  |  |
| Atmospheric Dispersion | mas | 0.59 |  |  |
| Residual flexure during exposure | mas | 1.56 |  |  |
| Contingency | mas | 0.59 |  |  |
| **Total image motion** | mas | 1.56 |  |  |

**Performance Metrics**

| Strehl ratio |  |  |  | SCI-1883 |
| FWHM | mas | 10.0 | 13.5 | 17.0 | 28.0 | 38.0 |

Figure 10: GLAO top-down error budget for SCI-1887 Requirement.
4.2. GLAO

For seeing-limited AO (GLAO), it is more appropriate to track the contribution to the FWHM, or to sum the structure functions. We choose to track the error budget in terms of contribution to FWHM of the average image over the FOV of 6.5 arcmin diameter. The values are calculated, where applicable for K band observations.

The error budget for the GLAO System presented here considers two main components also: High order and Image Motions Errors.

4.2.1. High Order Errors

- **GLAO High Order Error**: The high Order Error will be composed of several components, some of them are described below:
  
  - The WFS will measure the wavefront with associated noise due to the brightness of the guide star and other sources. We assume the guide stars can be made sufficiently bright (Using Laser guidestars) to make this error term negligible compared to other sources, and that the level of flux needed is approximately 2000 photons/m²/s. This condition will be confirmed with simulations.
  
  - The lag of the GLAO System in correcting wavefront error will add to the image width, but it is expected that the slower evolution of the ground layer, and the ability to provide a bright guide star constellation will allow us to minimize this error relative to other terms.
  
  - The LGS spot elongation due to the cone effect will contribute to the wavefront measurement error, and may be significant near the edge of the aperture.
  
  - The ASM is conjugated to a height 190 m above the primary mirror. The contributions from above and below this conjugate height will not be corrected as well as the fraction at 190 m.
  
  - The wavefront fitting is limited by the subaperture size of the WFS. The effect is more pronounced for short wavelengths, where the Fried length is shorter. At K band, the effect is relatively small for subapertures below 1 m.

- **GLAO Static Calibration**: These are the contributions by the NCPA of the system, these include:
  
  - WFS optics
  - GLAO dichroic
  - Alignment (e.g. WFS-to-ASM)
  - Field-dependent aberrations

- **Uncorrectable Telescope Aberrations**: These are the contributions of the residual wavefront error that the telescope will be able to perform w/o the AO.

- **Uncorrectable Instrument Aberrations**: These are the contributions of the residual wavefront error that the instruments because of their optics and design will contribute to the overall image quality

- **Uncorrectable Upper Layer contributions**: The upper layer term defines the fundamental limit to the image correction. Although the free atmosphere, as tabulated in Tokovin [4], is approximately 430 mas, some of this is corrected in the GLAO System. This term is primarily a comparison for the rest of the error budget to understand whether the design or the upper atmosphere is limiting our correction. To this end the values adopted are better than median conditions, to ensure the system design does not limit the GLAO performance.

4.2.2. Image Motions Errors

- **GLAO Tip/Tilt Errors**: The tip-tilt sensing will be carried out significantly off-axis from the science field. Typical guide stars will be 8-10' from the science field. This will contribute to the image width due to the guidestars being outside the atmospheric isokinetic patch center on the science field. The GLAO requirements can be met with a system, which delivers <0.26 arcsec RMS high-order wavefront error at the science detector, and <0.03 arcsec of residual tip-tilt error averaged in 6.5 arcmin.

  Figure 10 summarizes the terms described above for the GLAO System for a specific science requirement SCI-1887.
5. GMT ADAPTIVE OPTICS SYSTEM BOTTOM-UP ERROR BUDGET

Once initial error budgets have been generated by allocation, it is necessary to conduct analysis and simulations to derive the design parameters that will enable the AO System to deliver this performance in the specified conditions.

GMT AO primarily uses two analysis and simulation tools to explore the optimal design that meets the high-level error budget for each mode. These are:

- The Object Oriented Matlab Adaptive Optics (OOMAO, [6]) modeling library is a library of Matlab classes developed for the purpose of facilitating a clear, accessible end-to-end model of AO System. Objects from the different classes are assembled to perform numerical modeling of an AO System. OOMAO can be seen as an extension of the Matlab language; overloaded Matlab operators are used to propagate the wavefront through the system and to update the status of each object. The source object propagates a wavefront, both amplitude and phase, through the different objects representing the atmosphere, the telescope, the wavefront sensor, etc. Both NGS and LGS asterisms can be simulated. OOMAO can implement both open-loop and closed-loop systems and has been specifically designed for the modeling tomographic AO System.

- YAO (Yorick Adaptive Optics) is a general-purpose AO simulation tool written in Yorick, a fast scripting language. YAO has been and continues to be developed and extended for use with GMT. The software is configured for a particular scenario by modifying parameter files rather than changing any code.

Full end-to-end simulations on a 25 m telescope can take hours to run. In order to explore parameter space and understand trends associated with each parameter, the project has often used analytical equations from established sources. However, while the trend is usually correct, the amplitude of the effect can be incorrect by a significant amount.

Once parameters have been explored, a set of full end-to-end simulations is run on a subset of parameters close to the analytically derived optimal values. Isolation of a particular error term from these end-to-end simulations can be time consuming as it requires the end-to-end simulation to be re-run with certain aspects modified to estimate their contribution to the total error.

5.1. Worked Example - LTAO Atmospheric Fitting Error

A recent example of the bottom-up process that has encountered the issues discussed in the previous section is the LTAO High Order Atmospheric Fitting error term.

The assigned value of 105 nm was derived using the following analytical formula where the wavefront sensor is modeled as an ideal high-pass filter canceling all spatial frequency components in the wavefront up to the wavefront sensor cut-off frequency \( f_c = 1/2 \alpha \)

\[
\sigma_e^2 = \sigma_\phi^2 - \int_{-f_c}^{f_c} d\phi d f \phi(\alpha(f))
\]

Where \( \phi(\alpha(f)) \) is the power spectrum of the turbulence phase written:

\[
\phi(\alpha(f)) = 0.0229r_0^{-5/3} \left( f^2 + \frac{1}{L_0} \right)^{-11/6}
\]

We assume, that the fitting would be limited by the LTAO WFS Shack-Hartmann fitting, rather than the ASM actuator spacing. The outer scale, \( L_0 \), is assumed to be 60 m. The adopted subaperture size of 50 cm is derived from analytical calculations of signal to noise ratios due to shot noise in the detection of photons from the laser guidestars.

It should be noted that the above equation does not take into account the segmentation of the GMT pupil. As the design progressed and the assumptions were analyzed in greater detail, the number of photon detection events was revised up, based on a better understanding of the expected photon return from the sodium layer for a given laser format and power, a more accurate estimate of the throughput of the telescope and AOS optics and higher quantum efficiency for the detector. In addition, the read noise of the detector was known. The expected signal to noise in a subaperture was then found to be larger than previously predicted. The subaperture size could therefore be decreased.
Determining the optimum subaperture size required end-to-end simulations that included all of the above parameters but also took into account effects such as spot elongation.

Figure 11: End-to-end simulations of the GMT LTAO Mode: the predicted WFE is plotted as a function of the number of subapertures sampling the GMT pupil in the Shack-Hartmann wavefront sensor.

Figure 11, from Conan [8], shows the results of analytical analysis and end-to-end simulations using OOMAO. The blue line (Fit. + Tom.) is an analytically determined wavefront error and includes the High Order Atmospheric Fitting and Tomography error terms only. At 50x50 subapertures (50 cm) it is equal to the RSS of the error budget of these two terms.

The green line (noise free (non-elongated)) is determined by end-to-end simulations and includes the Atmospheric Fitting, Temporal Bandwidth and Tomography error terms. There are six LGS at a constellation radius of 30 arcsec and the system is running at 500 Hz. However, there is no noise and there is no spot elongation. This curve follows the analytical (blue line) however it is significantly higher than what would be predicted by the RSS addition of the Temporal Bandwidth error term alone.

The red line (noise free (elongated)) includes spot elongation from the laser being launched from three different locations at the edge of the primary mirrors, approximately 12 m off-axis. This line shows that the optimal number of apertures is around 60x60 subapertures (42 cm).

The simulations were then repeated with the addition of noise and a photon return of 349ph/cm2/s to see if this had any significant impact on the optimum subaperture size. It can be seen in Figure 11 that there is no significant change in the optimum value. The wavefront error for the included terms is less than 194 nm as required by the RSS of the included high-order error terms.

The end-to-end simulation model was then used to isolate the Atmospheric Fitting error term for the 42 cm subapertures. One hundred independent phase screens were generated. The phase screens follow the atmosphere statistics defined in the “typical-typical” model for the GMT. Using the truncated pseudo-inverse of the poke matrix, each wavefront was reconstructed. The fitting wavefront error associated to each lenslet array was computed as the standard deviation over the GMT pupil of the difference between the original and the reconstructed phase screens. This technique leads to a mean wavefront reconstruction error of ~150 nm. This is significantly higher than predicted by the analytical formula, which would have been approximately 90 nm for 42 cm subapertures. However, it does partially explain the larger than expected difference between the analytical results (blue line) and the end-to-end simulation results (green line).

The main contribution to the difference between the analytical predicted error and the simulation results is due to the increased fidelity of the end-to-end simulation. Whereas the analytical approach modeled the system as an ideal filter, the end-to-end simulation includes limitations in the spatial frequency response of the system when using a modal reconstruction of the wavefront. It also includes effects resulting from the segmented GMT pupil, including increased
edge effects, and errors in the pseudo-inverse of the poke matrix (i.e. degenerated eigen-modes with low eigen-values induced by the segmented shape of the pupil). The above analysis shows that while the RSS of included error terms is less than the requirement, the distribution of the contribution to wavefront error needs to be re-balanced. This will occur as further terms are isolated and analyzed and as a result, we will need to rebalance the error budgets accordantly.

More examples of bottom-up analysis for the GMT AO System are shown in Conan [8], van Dam [10], and Esposito[11].

6. PLAN FOR RESOLVING DISCREPACIES

6.1. Trades in subsystem requirements
A trade study (or trade-off study) is a formal tool that supports decision-making and it is an objective comparison of alternative solutions with respect to performance, cost, schedule, risk, and all other reasonable criteria. The systems engineering method relies on making design decisions using the trade studies. Trade studies are necessary when the system is complex and there is more than one design approach.

As the design progress and the analysis are better understood, several trades studies will be performed to help in the subsystem design. More about GMT AO System trade studies are described in Conan [8], Hinz [9], van Dam [10] and Esposito [11].

6.2. Monitoring Performance Related Subsystem errors
GMT has chosen the Cognition Cockpit Requirement Management Software [13] as the GMT Project requirement management tool. It is a web-based database that allows connecting requirements flow downs and budgets to be connected in a simple and effective way. An overview of the GMTO implementation of Cockpit is in Maiten [12].

![Requirement overview](image)

Figure 12: Requirement overview.

GMTO requirements will be divided in two major types, Pass/Fail or Numeric. Specifying a requirement type as numeric is seen to be advantageous when the flow down of high level requirements is quantified by transfer functions (Figure 12). The transfer functions facilitate the roll up of nominal and variation values which then allows Cockpit to compute and report the satisfaction of a requirement using statistical metrics.

The satisfaction metrics of a requirement are also color-coded, marking requirements that are satisfied and complete as
green, the requirements that are incomplete as grey, the ones that are not satisfied as yellow or red depending on the value of the satisfaction metric and the requirements that are overdesigned as blue. The AO System engineer can monitor these metrics using the Cockpit Tiered Scorecards (Figure 13).

For instance, a Science Requirement (SCI-1885) will break down into System level requirements that then flow down into all of the subsystems (AO, TEL, ...etc.), using the error budgets as the Top flow down requirements (Figure 15).
For each of these child requirements, when they are defined as numeric and quantified by transfer function models (error budget analytic formulas from MS Excel models), Cockpit is able to calculate and rollup nominal and variation values when given the formula for how the requirements interact.

Now by comparing the design (process or test values) against the required target value, Cockpit computes and reports on how well the science requirement has been satisfied (Figure 14).

The difficult aspect of this process is to incorporate the correct formulas, which need to take in account the potentially non-independent nature of errors budget terms.

To minimize risks associated with meeting the science requirements, continual monitoring of the design, fabrication, and integration progress will be needed to update the error budget as the system evolves. This will be accomplished by reviews on a regular basis of any interface related issues. These reviews will monitor any changes of any expected subsystem performance estimates. We will track all this information in Cockpit and we will use the Cockpit report to track down inconsistencies.

AO System engineering will be responsible for tracking of the overall AO error budget and updating it from a bottom-up approach as needed. Specifically, any problem areas will be identified and tracked until a satisfactory solution is found. Inputs from the responsible engineering areas to this effort will be required along with help to resolve any issues. Major updates/reviews of the error budget should coincide with major design, fabrication, and integration milestones.

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